SAINT DOMINIC ACADEMY
MATHEMATICS DEPARTMENT

ENTERING AP CALCULUS AB

2024 SUMMER PACKET

DUE ON THE FIRST DAY OF SCHOOL

D I R E C T I O N S
Join the Google Classroom. The code is **7sw3hri**
Solve all problems. Show all necessary and complete work in PENCIL.
Write legibly and as neatly as possible.

Cheating is prohibited.

CALCULATOR IS NOT ALLOWED

Name: _____   Signature: _____

AP CALCULUS AB
Summer Assignment

Name: _____

Going into AP Calculus, there are certain skills that have been taught to you over the previous years that we assume you have. If you do not have these skills, you will find that you will consistently get problems incorrect next year, even though you understand the calculus concepts. It is frustrating for students when they are tripped up by the algebra and not the calculus. This summer packet is intended for you to brush up and possibly relearn these topics.

I assume that you have basic skills in algebra. Being able to solve equations, work with algebraic expressions, and basic factoring, for example should now be a part of you. Only the topics I see that students consistently have difficulty with that are continually used in AP Calculus are included here.

Here are some useful websites:
http://www.purplemath.com/modules/index.htm
http://www.khanacademy.org/#browse

On the following 15 pages, you have 9 to 12 problems per page. Each problem should be done in the space provided. If and when you're unsure of how to attempt these problems, examine the suggested websites.

Realize also that certain concepts are interrelated. Domain, for example, may require you to be expert at working with inequalities. Solving quadratic equations may involve techniques used in solving fractional equations.

This packet is due on the first day of school in September. It will be graded. You need to get off to a good start so spend some quality time on this packet this summer. Work needs to be shown when needed. Also do not rely on the calculator. Half of your AP exam next year is taken without the use of a calculator. So paper and pencil techniques only.

It is a mistake to decide to do this now. Let it go until mid-summer. I want these techniques to be relatively fresh in your mind in the fall. Also, do not wait to do them at the very last minute. These take time. If you do two concepts a day, the whole packet will take you about a week to complete.

If you have questions about any of these problems, contact me at the school email address. Have a good summer and see you in the fall.

The topics are listed below.
Beginning Algebra topics
    1. Fractional and Negative Exponents

Intermediate Algebra Topics
2. Domain
3. Solving Inequalities: Absolute Value
4. Solving Inequalities: Quadratic
5. Special Factoring Formulas
6. Function Transformations
7. Factor Theorem (p over q/synthetic division)
8. Even and Odd Functions
9. Solving Quadratic Equations

Advanced Algebra Topics
10. Asymptotes
11. Complex Fractions
12. Composition of Functions
13. Solving Rational(Fractional) Equations

Trigonometry Information
14. Basic Right Angle Trigonometry
15. Solving Trigonometric Equations

Be safe, be motivated, do the right thing. Enjoy your summer!

Mrs. Patiak

Name_____

Topic 1: Fractional & Negative Exponents

Simplify using only positive exponents

1. $-3x^{-3}$

2. $-5\left(\dfrac{3}{2}\right)(4-9x)^{\frac{-1}{2}}(-9)$

3. $2\left(\dfrac{2}{2-x}\right)\left[\dfrac{-2}{(2-x)^2}\right]$

4. $\left(16x^2y\right)^{\frac{3}{4}}$

5. $-\dfrac{x^{\frac{-1}{2}}}{2}\sin\sqrt{x}$

6. $\dfrac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

7. $-4\left(\dfrac{2x-1}{2x+1}\right)^{-3}\left[\dfrac{2(2x+1)-2(2x-1)}{(2x+1)^2}\right]$

8. $\dfrac{\dfrac{1}{2}(2x+5)^{-\frac{3}{2}}}{\dfrac{3}{2}}$

9. $\left(\dfrac{1}{x^{-2}}+\dfrac{4}{x^{-1}y^{-1}}+\dfrac{1}{y^{-2}}\right)^{\frac{-1}{2}}$

Topic 2: Domain

Find the domain of the following functions:

1. $y = \dfrac{3x - 2}{4x + 1}$

2. $y = \dfrac{x^2 - 4}{2x + 4}$

3. $y = \dfrac{x^2 - 5x - 6}{x^2 - 3x - 18}$

4. $y = \dfrac{2^{2-x}}{x}$

5. $y = \sqrt{x - 3} - \sqrt{x + 3}$

6. $y = \dfrac{\sqrt{2x - 9}}{2x + 9}$

7. $y = \dfrac{x^2 + 8x + 12}{\sqrt[4]{x + 5}}$

8. $y = \sqrt{x^2 - 5x - 14}$

9. $y = \dfrac{\sqrt[3]{x - 6}}{\sqrt{x^2 - x - 30}}$

10. $y = \log(2x - 12)$

11. $y = \sqrt{\tan x}$

12. $y = \dfrac{x}{\cos x}$

Topic 3: Solving inequalities (absolute value)

Write the following absolute value expressions as piecewise expressions

1. $y = |2x - 4|$

2. $y = |6 + 2x| + 1$

3. $y = |4x + 1| + 2x - 3$

Solve the following absolute value inequalities

4. $|x - 3| > 12$

5. $|x - 3| \leq 4$

6. $|10x + 8| > 2$

7. $|3x - 4| > -2$

8. $|x - 6| > -8$

9. $|x + 1| \leq |x - 3|$

Topic 4: Solving inequalities (quadratic)

Write the following absolute value expressions as piecewise expressions

1. $\left| x^2 - 1 \right|$

2. $\left| x^2 + x - 12 \right|$

3. $\left| x^2 + 4x + 4 \right|$

Solve the following by factoring and making appropriate sign charts.

4. $x^2 - 16 > 0$

5. $x^2 + 6x - 16 > 0$

6. $x^2 - 3x \geq 10$

7. $2x^2 + 4x \leq 3$

8. $x^3 + 4x^2 - x \geq 4$

9. $2\sin^2 x \geq \sin x \qquad 0 \leq x < 2\pi$

Topic 5: Special factorization

Factor completely

1. $x^3 + 8$

2. $x^3 - 8$

3. $27x^3 - 125y^3$

4. $x^4 + 11x^2 - 80$

5. $ac + cd - ab - bd$

6. $2x^2 + 50y^2 - 20xy$

7. $x^2 + 12x + 36 - 9y^2$

8. $x^3 - xy^2 + x^2y - y^3$

9. $(x-3)^2(2x+1)^3 + (x-3)^3(2x+1)^2$

Topic 6: Function transformation

If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$:
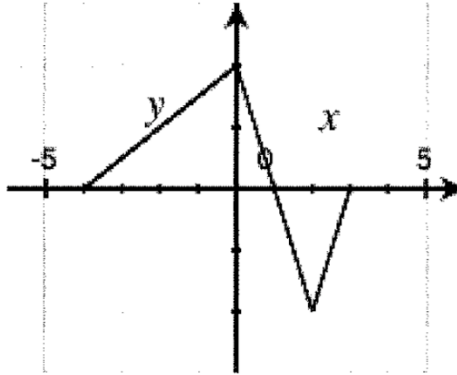
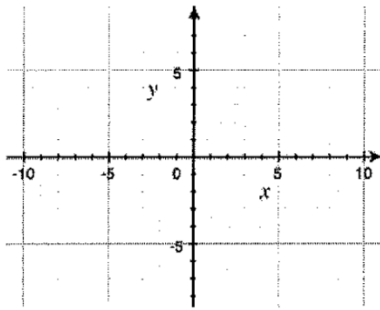1. $f(x) - 4$

2. $f(x - 4)$
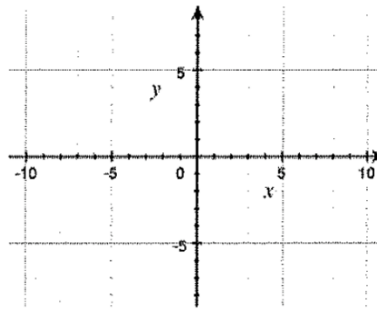
3. $-f(x + 2)$

4. $5f(x) + 3$

5. $f(2x)$

6. $|f(x)|$

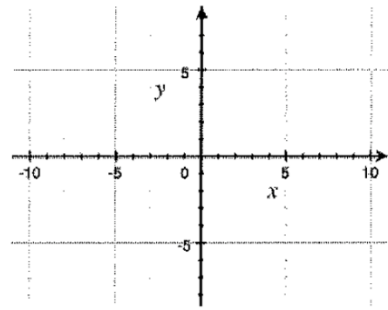Here is a graph of $y = f(x)$. Sketch the following graphs
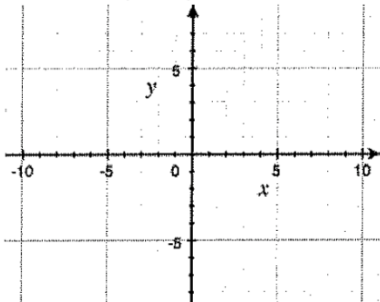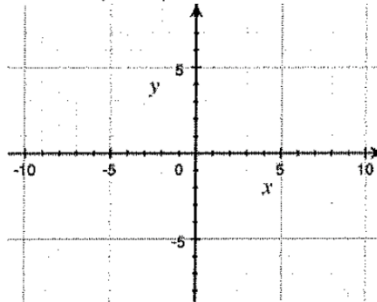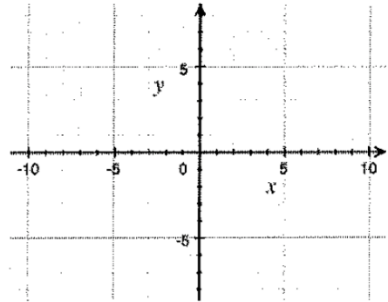

7. $y = 2f(x)$

8. $y = -f(x)$

9. $y = f(x - 1)$

10. $y = f(x + 2)$

11. $y = |f(x)|$

12. $y = f|x|$

Topic 7: Factor theorem (*p* over *q* method/synthetic division)

Use the *p* over *q* method and synthetic division to factor the polynomial $P(x)$. Then solve $P(x) = 0$.

1. $P(x) = x^3 + 4x^2 + x - 6$

2. $P(x) = x^3 + 5x^2 - 2x - 24$

3. $P(x) = x^3 - 6x^2 + 3x - 10$

4. $P(x) = x^3 + 2x^2 - 19x - 20$

5. $P(x) = x^4 + 5x^3 + 6x^2 - 4x - 8$

6. $P(x) = x^4 + 11x^3 + 41x^2 + 61x + 30$

Topic 8: Even and odd functions

Show work to determine if the relation is even, odd, or neither

1. $f(x) = 2x^2 - 7$

2. $f(x) = -4x^3 - 2x$

3. $f(x) = 4x^2 - 4x + 4$

4. $f(x) = x - \dfrac{1}{x}$

5. $f|x| = |x| - x^2 + 1$

6. $5x^2 - 6y = 1$

7. $y = e^x - \dfrac{1}{e^x}$

8. $3y^3 = 4x^3 + 1$

9. $3x = |y|$

Topic 9: Solving quadratic equations

Solve each equation

1. $7x^2 - 3x = 0$

2. $4x(x - 2) - 5x(x - 1) = 2$

3. $x^2 + 6x + 4 = 0$

4. $2x^2 - 3x + 3 = 0$

5. $2x^2 - (x + 2)(x - 3) = 12$

6. $x + \dfrac{1}{x} = \dfrac{13}{6}$

7. $x^4 - 9x^2 + 8 = 0$

8. $x - 10\sqrt{x} + 9 = 0$

9. $\dfrac{1}{x^2} - \dfrac{1}{x} = 6$

Topic 10: Asymptotes

For each function, find the equations of both the vertical asymptote(s) and horizontal asymptotes (if they exist)

1. $y = \dfrac{x}{x-3}$

2. $y = \dfrac{x+4}{x^2-1}$

3. $y = \dfrac{x+4}{x^2+1}$

4. $y = \dfrac{x^2-2x+1}{x^2-3x-4}$

5. $y = \dfrac{x^2-9}{x^3+3x^2-18x}$

6. $y = \dfrac{2x^2+6x}{x^3-3x^2-4x}$

7. $y = \dfrac{x^2-x-6}{x^3-x^2+x-6}$

8. $y = \dfrac{2x^3}{x^3-1}$

9. $y = \dfrac{\sqrt{x}}{2x^2-10}$

Topic 11: Complex Fractions

Simplify each of the following expressions.

1. $\dfrac{2a - \dfrac{1}{8a}}{4 + \dfrac{1}{a}}$

2. $\dfrac{1}{2 - \dfrac{1}{m-4}}$

3. $\dfrac{1}{1 - \dfrac{1}{x-1}}$

4. $\dfrac{\dfrac{1}{a} + \dfrac{1}{b}}{\dfrac{1}{a} - \dfrac{1}{b}}$

5. $\dfrac{5 - \dfrac{x}{x-1}}{2 - \dfrac{x}{x+1}}$

6. $\dfrac{\dfrac{1}{x+a} + \dfrac{1}{x-a}}{\dfrac{1}{x+a} - \dfrac{1}{x-a}}$

7. $1 - \dfrac{1}{1 - \dfrac{1}{1 - \dfrac{1}{q-1}}}$

8. $\dfrac{6 + \dfrac{2}{x}}{\dfrac{3x+1}{4}}$

9. $\dfrac{2}{\dfrac{1}{a} + \dfrac{1}{b}}$

Topic 12:   Composition of functions

If $f(x) = x^2$, $g(x) = 2x - 1$, and $h(x) = 2^x$, find the following

1. $f(g(2))$

2.. $f(g(2))$

3. $f(h(-1))$

4. $h(f(-1))$

5. $g\left(f\left(h\left(\dfrac{1}{2}\right)\right)\right)$

6. $f(g(x))$

7. $g(f(x))$

8. $g(g(x))$

9. $f(h(x))$

Topic 13: Solving Rational (fractional) equations

Solve each equation for x

1. $\dfrac{2}{3} - \dfrac{5}{6} = \dfrac{1}{x}$

2. $x + \dfrac{6}{x} = 5$

3. $\dfrac{x+1}{3} - \dfrac{x-1}{2} = 1$

4. $\dfrac{x-5}{x+1} = \dfrac{3}{5}$

5. $\dfrac{60}{x} - \dfrac{60}{x-5} = \dfrac{2}{x}$

6. $\dfrac{2}{x+5} + \dfrac{1}{x-5} = \dfrac{16}{x^2-25}$

7. $\dfrac{x}{x-2} + \dfrac{2x}{4-x^2} = \dfrac{5}{x+2}$

8. $\dfrac{x}{2x-6} - \dfrac{3}{x^2-6x+9} = \dfrac{x-2}{3x-9}$

9. $\dfrac{2x+3}{x-1} = \dfrac{10}{x^2-1} + \dfrac{2x-3}{x+1}$

Topic 14: Solving Rational (fractional) equations

Solve the following problems.

If point P is on the terminal side of θ, find all 6 trig functions of θ. Draw a picture.

1. $P(-2,4)$

2. $P(\sqrt{5},-2)$

3. If $\cos\theta = \dfrac{5}{13}, \theta$ in quadrant II,
   find $\sin\theta$ and $\tan\theta$

4. If $\cot\theta = 3, \theta$ in quadrant III,
   find $\sin\theta$ and $\cos\theta$

Find the exact value of the following without calculators:

5. $\sin^2 225° - \cos^2 300°$

6. $\left(6\sec180° - 4\cot90°\right)^2$

7. $\left(4\cos30° - 6\sin120°\right)^{-2}$

Solve the following triangles (3 decimal place accuracy)

8. $A =$ $a = 21.7$
   $B = 16°$ $b =$
   $C = 90°$ $c =$

9. $A =$ $a = 6$ feet
   $B =$ $b =$
   $C = 90°$ $c = 95$ inches

Topic 15: Solving Trigonometric equations

Solve each equation on the interval $[0, 2\pi)$

1. $\sin x = \dfrac{1}{2}$

2. $\cos^2 x = \cos x$

3. $2\cos x + \sqrt{3} = 0$

4. $4\sin^2 x = 1$

5. $2\sin^2 x + \sin x = 1$

6. $\cos^2 x + 2\cos x = 3$

7. $2\sin x \cos x + \sin x = 0$

8. $8\cos^2 x - 2\cos x = 1$

9. $\sin^2 x - \cos^2 x = 0$