

Algebra I - Summer Review Packet - 2022

DUE AUGUST 31, 2022 in the appropriate Google Classroom

Welcome to Saint Dominic Academy! We are glad you are here with us. In preparation for the Fall Semester, this assignment is required to prepare you for your Algebra 1 Course.

A <u>**TI-84 Plus CE</u>** graphing calculator is recommended for this course. Most teachers at Saint Dominic Academy use the TI-84. Students who purchase a difference model (e.g. TI-Nspire, Casio, etc) will be responsible for learning their operation.</u>

About Algebra I:

Algebra I teaches students to think, reason, and communicate mathematically. Students use variables to determine solutions to real world problems. Skills gained in Algebra I provide students with a foundation for subsequent math courses. Students use a graphing calculator as an integral tool in analyzing data and modeling functions to represent real world applications. Each student is expected to use calculators in class, on homework, during tests, and during midterm and final exams. You will be able to use this calculator for your four years at Saint Dominic Academy Academy and beyond. Calculators can be purchased on-line and in many department stores, i.e., Target.

Expectations of the Summer Packet:

The problems in this packet are designed to help you review topics that are important to your success in Algebra I. *All work must be shown for each problem*. The problems should be done correctly, not just attempted. Don't forget to check your work for problems when solutions can be checked.

All work should be completed and ready to turn in to the Google Classroom.

There may be a **QUIZ** on this material at the beginning of school.

Notes: The internet is a great resource... use it! Some helpful sites: <u>www.purplemath.com/modules/index.html</u> <u>www.youtube.com</u> <u>www.khanacademy.com</u>

Enjoy your summer!

I. Adding and Subtracting Integers

Adding Integers	Add their absolute values. The sum is:
with the Same Sign	• positive if both integers are positive.
in the came e.g.	 negative if both integers are negative.

Example 1	Find the sum $-3 + (-4)$.
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-3 + (-4) = -7 Add |-3| and |-4|. The sum is negative.

Adding Integers with Different Signs	Subtract their absolute values. The sum is: • positive if the positive integer's absolute value is greater. • negative if the negative integer's absolute value is greater.	
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Subtracting	To subtract an integer, add its additive inverse.
Integers	

Example 1 Find each difference.

a. 9 – 17 b. –7 – 3			
9 - 17 = 9 + (-17)	To subtract 17, add –17.	-7 - 3 = -7 + (-3)	To subtract 3, add -3.
= -8	Simplify.	= -10	Simplify.

Example 2	Find each difference.
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a. 4 – (–5)		b. $-6 - (-2)$	
4 - (-5) = 4 + 5	To subtract -5, add +5.	-6 - (-2) = -6 + 2	To subtract -2, add +2.
= 9	Simplify.	= -4	Simplify.

Find each sum or difference.

-12 + (-15)

3. 9 + (-25) **4**. 8 - (-6)

5 15 – (-25)	6 13 – 17
	0. 10 1/

II. Multiplying and Dividing Integers

Multiplying Integers with Different Signs	The product of two integers with different signs is negative.	
Example 1 Find ea	ach product.	
a. 4(-3)	b. -8(5)	
4(-3) = -12	-8(5) = -40	
Multiplying Integerswith the Same Sign		
Example 2 Find ea	ach product.	
a. 6(6)	b. -7(-4)	
6(6) = 36	-7(-4) = 28	
Dividing Integers with the Same SignThe quotient of two integers with the same sign is positive.		
Example 1Find each quotient.a. $14 \div 2$ The dividend and the divisor have the same sign. $14 \div 2 = 7$ The quotient is positive.		
b. $\frac{-25}{-5}$ $\frac{-25}{-5} = -25 \div (-5)$ = 5	The dividend and divisor have the same sign. The quotient is positive.	
Dividing Integers with Different Signs	The quotient of two integers with different signs is negative.	

Find each product or quotient.

1 . 12 (-6)	2 10(4)
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III. Writing Algebraic Expressions

In **algebraic expressions**, letters such as *x* and *w* are called variables. A variable is used to represent an unspecified number or value. Practice: Write an algebraic expression for each verbal expression.

1. Four times a number decreased by twelve _____

2. Three more than the product of five and a number _____

3. The quotient of two more than a number and eight _____

4. Seven less than twice a number ______

IV. Order of Operations

To evaluate numerical expressions containing more than one operation, use the rules for order of operations. The rules are often summarized using the expression **PEMDAS**

Examples:

Parentheses (Grouping Symbols)	$[(7-4)^2+3]+15$	$\frac{(9-7)^2+6}{11-6}$	
Exponents	= [3 ² + 3] + 15	$=\frac{2^2+6}{5}$	
Multiply or Divide, from left to right	= [9+3]+15	$=\frac{4+6}{5}$	
Add or Subtract, from left to right	= 12 + 15	$=\frac{10}{2}$	
		= 5	

Evaluate each expression.

1.
$$250 \div [5(3 \cdot 7 + 4)]$$
 2. $\frac{5^2 \cdot 4 - 5 \cdot 4^2}{5(4)}$

3.
$$\frac{1}{2} \cdot 26 - 3^2$$

4. $8^2 \div (2 \cdot 8) + 2$

5.
$$5 + [30 - (6 - 1)^2]$$

6. $\frac{2 \cdot 4^2 - 8 \div 2}{(5 + 2) \cdot 2}$

V. Evaluating Algebraic Expressions

To evaluate algebraic expressions, first replace the variables with their values. Then, use order of operations to calculate the value of the resulting numerical expression.

Example: Evaluate $x^2 - 5(x - y)$ if x = 6 and y = 2

$$x^{2} - 5(x - y) = (6)^{2} - 5(6 - 2)$$

= (6)^{2} - 5(4)
= 36 - 5(4)
= 36 - 20
= 16

Evaluate each expression.

1.
$$5x^2 - y$$
 when $x = 4$ and $y = 24$
2. $\frac{3xy-4}{7x}$ when $x = 2$ and $y = 3$

3.
$$(z \div x)^2 + \frac{4}{5}x$$
 when $x = 2$ and $y = 4$
4. $\frac{y^2 - 2z^2}{x + y - z}$ when $x = 12, y = 9$ and $z = 4$

VI. The Real Number System

Real Numbers					
Rational $\frac{5}{3}$ 0.63 0.012	Irrational				
Integers {, -2, -1, 0, 1, 2,}	$\sqrt{3}$ π 0.10010001				
Whole {0, 1, 2, 3,}					
Natural {1, 2, 3,}					

The **Real** number system is made up of two main sub-groups **Rational numbers** and **Irrational numbers**.

The set of rational numbers includes several subsets: **natural numbers, whole numbers, and integers.**

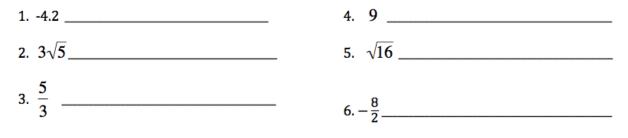
- Real Numbers- any number that can be represented on a number-line.
 - Rational Numbers- a number that can be written as the ratio of two integers (this includes decimals that have a definite end or repeating pattern)

Examples: 2, $-5, \frac{-3}{2}, \frac{1}{3}, 0.253, 0.\overline{3}$

- Integers- positive and negative whole numbers and 0 Examples: -5, -3, 0, 8 ...
- Whole Numbers the counting numbers from 0 to infinity Examples: { 0, 1, 2, 3, 4,}
- Natural Numbers- the counting numbers from 1 to infinity Examples: { 1, 2, 3, 4... }
- Irrational Numbers- Non-terminating, non-repeating decimals (including π, and the square root of any number that is not a perfect square.)

Examples: 2π , $\sqrt{3}$, $\sqrt{23}$, 3.21211211121111...

Name all the sets to which each number belongs.



VII. The Distributive Property

The Distributive Property states any number a, b, and c: 1. a(b + c) = ab + ac or (b + c)a = ba + ca

2.
$$a(b - c) = ab - ac$$
 or $(b - c)a = ba - ca$

Rewrite ach expression using the distributive propery.

1.
$$7(h - 3)$$
 2. $-3(2x + 5)$

3.
$$(5x - 9)4$$
 4. $\frac{1}{2}(14 - 6y)$

5.
$$3(7x^2 - 3x + 2)$$
 6. $\frac{1}{4}(16x - 12y + 4z)$

7.
$$(9 - 2x + 3xy) \cdot -4$$

8. $0.3(40a + 10b - 5)$

VIII. Combining Like Terms

Terms in algebra are numbers, variables or the product of numbers and variables. In algebraic expressions terms are separated by addition (+) or subtraction (-) symbols. Terms can be combined using addition and subtraction if they are **like-terms**.

Like-terms have the same variables to the same power. Example of like-terms: $5x^2$ and $-6x^2$

> Example of terms that are **NOT** like-terms: $9x^2$ and 15xAlthough both terms have the variable **x**, they are not being raised to the same power

To combine like-terms using addition and subtraction, add or subtract the numerical factor Example: Simplify the expression by combining like-terms

 $8x^{2} + 9x - 12x + 7x^{2} = (8+7)x^{2} + (9-12)x$ $= 15x^{2} + -3x$ $= 15x^{2} - 3x$

Simplify each expression.

1. 5x - 9x + 2 **2.** $3x^2 + x - x^2$

3. $x^2 + 4x^2 - 7x^2$ **4.** $5x^2 + 6x - 12x^2 - 9x + 2$

5. 2(3x - 4y) + 5(x + 3y) **6.** $10xy - 4(xy + 2x^2y)$

IX. Solving Equations with Variables on One Side

To solve an equation means to *find the value* of the variable. We solve equations by isolating the variable using opposite operations.

Example: Solve.		Opposite Operations: Addition (+) & Subtraction (-)
3x - 2 = 10		Multiplication (x) & Division (+)
+2 +2	Isolate 3x by adding 2 to each side.	2
$\frac{3x}{3} = \frac{12}{3}$	Simplify Isolate x by dividing each side by 3.	Please remember to do the same step on each side of the equation.
x = 4 Check your answer.	Simplify	Always check your work by substitution!
3(4) - 2 = 10	Substitute the value in for the variable.	
12 - 2 = 10	Simplify	
10 = 10	Is the equation true? If yes, you sol	ved it correctly!

Solve each equation.

1. 98 = <i>b</i> + 34	2. −14 + <i>y</i> = −2
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3. 8 <i>k</i> = - 64	4. $\frac{2}{5}x = 6$
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5. 14 <i>n</i> – 8 = 34	6. 8 + $\frac{n}{12}$ = 13
J. 14 <i>11</i> = 0 = 34	$0.0 + \frac{1}{12} = 1$

X. Solving Equations with Variables on Each-Side:

To solve an equation with the same variable on each side, write an equivalent equation that has the variable on just one side of the equation. Then solve.

Example Solve 4(2a - 1) = -10(a - 5). 4(2a - 1) = -10(a - 5)Original equation 8a - 4 = -10a + 50**Distributive Property** 8a-4+10a=-10a+50+10a Add 10a to each side. 18a - 4 = 50Simplify. 18a - 4 + 4 = 50 + 4Add 4 to each side. 18a = 54Simplify. $\frac{18a}{18} = \frac{54}{18}$ Divide each side by 18. a = 3Simplify.

The solution is 3.

Solve each equation.

1.
$$5 + 3r = 5r - 19$$
 2. $8x + 12 = 4(3 + 2x)$

3.
$$-5x - 10 = 2 - (x + 4)$$
 4. $6(-3m + 1) = 5(-2m - 2)$

5. 3(d-8) - 5 = 9(d+2) + 1

XI. Ratios and Proportions

Solve Proportions If a proportion involves a variable, you can use cross products to solve the proportion. In the proportion $\frac{x}{5} = \frac{10}{13}$, x and 13 are called **extremes**. They are the first and last terms of the proportion. 5 and 10 are called **means**. They are the middle terms of the proportion. In a proportion, the product of the extremes is equal to the product of the means.

Means-Extr	emes Property of Proportions	For any numbers <i>a</i> , <i>b</i> , <i>c</i> , and <i>d</i> , if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.		
	Example 1: $\frac{x}{5} = \frac{10}{13}$ $x \cdot 13 = 5 \cdot 10$ 13x = 50 $\frac{13x}{13} = \frac{50}{13}$ $x = \frac{50}{13}$		Example 2: $\frac{x+1}{4} = \frac{3}{4}$ $4(x+1) = 3 \cdot 4$ $4x + 4 = 12$ $-4 - 4$ $4x = 8$ $\frac{4x}{4} = \frac{8}{4}$ $x = 2$	

Solve each proportion.

$$1. \frac{x}{21} = \frac{3}{63} \qquad \qquad 2. \frac{9}{x+1} = \frac{18}{54}$$

3.
$$\frac{-3}{x} = \frac{2}{8}$$
 4. $\frac{a-8}{12} = \frac{15}{3}$

5.
$$\frac{0.1}{2} = \frac{0.5}{x}$$
 6. $\frac{3+y}{4} = \frac{-y}{8}$

XII. Percent of Change

Percent of Change When an increase or decrease in an amount is expressed as a percent, the percent is called the **percent of change**. If the new number is greater than the original number, the percent of change is a **percent of increase**. If the new number is less than the original number, the percent of change is the **percent of decrease**.

Example 1 Find the percent of in original: 48 new: 60	ncrease.	Example 2 Find the perce original: 30 new: 22	ent of decrease.		
First, subtract to find the increase. The amount of $60 - 48 = 12$.		First, subtract to find the amount of decrease. The amount of decrease is $30 - 22 = 8$.			
Then find the percent of the original number, 48			ercent of decrease by using nber, 30, as the base.		
$\frac{12}{48} = \frac{r}{100}$ Percent	t proportion	$\frac{8}{30} = \frac{r}{100}$	Percent proportion		
12(100) = 48(r) Cross p	products	8(100) = 30(r)	Cross products		
1200 = 48r Simplify	<i>y</i> .	800 = 30r	Simplify.		
$\frac{1200}{48} = \frac{48r}{48}$ Divide	each side by 48.	$\frac{800}{30} = \frac{30r}{30}$	Divide each side by 30.		
25 = r Simplify	<i>y.</i>	$26\frac{2}{2} = r$	Simplify.		
The percent of increase	is 25%.	5	decrease is $26\frac{2}{3}\%$, or		

about 21%.

State whether each percent of change is a percent of increase or percent of decrease. Then find each percent of change.

1. original: 50 new: 80 **2.** original: 27.5 new: 25

3. original: 14.5 new: 20

4. original: 250 new: 500

XII. Percent of Change (Continued)

Solve Problems Discounted prices and prices including tax are applications of percent of change. Discount is the amount by which the regular price of an item is reduced. Thus, the discounted price is an example of percent of decrease. Sales tax is amount that is added to the cost of an item, so the price including tax is an example of percent of increase.

Example A coat is on sale for 25% off the original price. If the original price of the coat is \$75, what is the discounted price?

The discount is 25% of the original price.

The discounted price of the coat is \$56.25.

Find the final price of each item. When a discount and sales tax are listed, compute the discount price before computing the tax.

1. Two concert tickets: \$28 Student discount: 28% 4. Celebrity calendar: \$10.95 Sales tax: 7.5%

- Airline ticket: \$248.00 Frequent Flyer discount: 33%
- 5. Camera: \$110.95 Discount: 20% Sales tax: 5%

3. CD player: \$142.00 Sales tax: 5.5% Ipod: \$89.00 Discount: 17% Tax: 5%

XIII. Solving for a Specific Variable

Solve for Variables Sometimes you may want to solve an equation such as $V = \ell wh$ for one of its variables. For example, if you know the values of V, w, and h, then the equation $\ell = \frac{V}{wh}$ is more useful for finding the value of ℓ . If an equation that contains more than one variable is to be solved for a specific variable, use the properties of equality to isolate the specified variable on one side of the equation.

Example 1 Solve $2x - 4y = 8$ for y.	Example 2 Solve $3m - n = km - 8$ for <i>m</i> .
2x - 4y = 8 2x - 4y - 2x = 8 - 2x -4y = 8 - 2x $\frac{-4y}{-4} = \frac{8 - 2x}{-4}$ $y = \frac{8 - 2x}{-4} \text{ or } \frac{2x - 8}{4}$ The value of y is $\frac{2x - 8}{4}$.	3m - n = km - 8 3m - n - km = km - 8 - km 3m - n - km = -8 3m - n - km + n = -8 + n 3m - km = -8 + n m(3 - k) = -8 + n $\frac{m(3 - k)}{3 - k} = \frac{-8 + n}{3 - k}$ $m = \frac{-8 + n}{3 - k}, \text{ or } \frac{n - 8}{3 - k}$
	The value of <i>m</i> is $\frac{n-8}{3-k}$. Since division by 0 is
	undefined, $3 - k \neq 0$, or $k \neq 3$.

Solve each equation or formula for the variable specified.

1. 15x + 1 = for x **2.** 7x + 3y = for y

3. (4 - k) = for k

4. P = 2l + 2w for w

XIV. Rate of Change and Slope

Slope of a Line	$m = \frac{\text{rise}}{\text{run}}$ or $m = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are the coordinates of any two points on a nonvertical line
	of any two points of a nonvertical line

Example 1 Find the slope of the line that passes through (-3, 5) and (4, -2). Let $(-3, 5) = (x_1, y_1)$ and $(4, -2) = (x_2, y_2)$. $m = \frac{y_2 - y_1}{x_2 - x_1}$ Stope formula $= \frac{-2 - 5}{4 - (-3)}$ $y_2 = -2, y_1 = 5, x_2 = 4, x_1 = -3$ $= \frac{-7}{7}$ Simplify. = -1 Simplify. = -1 Simplify. = -1 Divide each side by -7.

Find the slope of the line that passes through each pair of points.

1.
$$(4,9), (1,-6)$$
 2. $(4,3.5), (-4,3.5)$

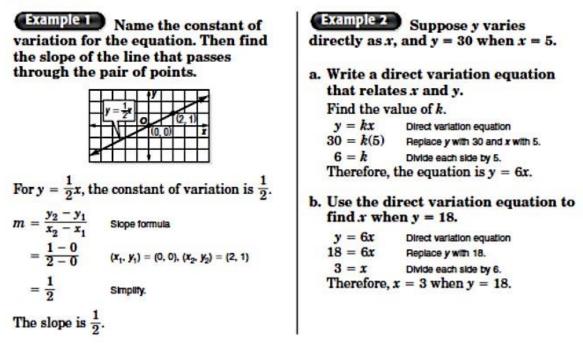
3.
$$(2, 5), (6, 2)$$
 4. $(1, -2), (-2, -5)$

Determine the value of *r* so the line that passes through ach pair of points has the given slope.

5. (6, 8),
$$(r, -2)$$
, $m = 1$
6. (10, r), (3, 4), $m = -\frac{2}{7}$

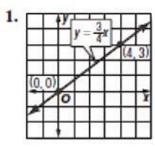
XV. Slope and Direct Variation

Direct Variation A direct variation is described by an equation of the form y = kx, where $k \neq 0$. We say that y varies directly as x. In the equation y = kx, k is the constant of variation.



Name the constant of variation for each equation. Then determine the slope for the line that passes through each pair of points.

2



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				(3,	4)	V	
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Write a direct variation equation that relates *x* and *y*. Assume that *y* varies directly as *x*. Then solve.

3. If y = 7.5 when x = 0.5, find *y* when x = -0.3.

4. If *y* = 80 when *x* = 32, find *x* when *y* = 100.

XVI. Linear Equations and Slope-Intercept Form

y - mx + b, where m is the given slope and b is the y-intercept Slope-Intercept Form Example 1 Write an equation of the line whose slope is -4 and whose y-intercept is 3. y = mx + bSlope-Intercept form Replace m with -4 and b with 3. y = -4x + 3Graph 3x - 4y = 8. 3x - 4y = 8Original equation -4y = -3x + 8Subtract 3x from each side. $\frac{-4y}{-4} = \frac{-3x+8}{-4}$ Divide each side by -4. $y=\frac{3}{4}x-2$ Simplify.

The y-intercept of $y = \frac{3}{4}x - 2$ is -2 and the slope is $\frac{3}{4}$. So graph the point (0, -2). From this point, move up 3 units and right 4 units. Draw a line passing through both points.

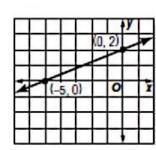
Write an equation of the line with the given slop and y-intercept.

1. slope: $\frac{1}{4}$, y-intercept: 3

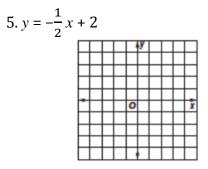
3.

2. slope: -2.5, y-intercept: 3.5

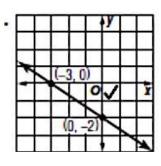
Write an equation of the line shown in each graph.



Graph each equation.



4.



6. 6x + 3y = 6

					y			
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XVII. Solving Word Problems

Write an algebraic equation to model each situation. Then solve the equation and answer the question.

1. A video store charges a one-time membership fee of \$11.75 plus \$1.50 per video rental. How many videos did Stewart rent if he spends \$72.00?

2. Darel went to the mall and spent \$41. He bought several t-shirts that ach cost \$12 and he bought 1 pair of socks for \$5. How many t-shirts did Darel buy?

3. Nick is 30 years less than 3 times Ray's age. If the sum of their ages is 74, how old are each of the men?

4. Three-fourths of the student body attended the pep-rally. If there were 1230 students at the pep rally, how many students are there in all?

5. Sarah drove 3 hours more than Michael on their trip to Texas. If the trip took 37 hours, how long did Sarah and Michael each drive?

6. Bicycle city makes custom bicycles. They charge \$160 plus \$80 for each day that it takes to build the bicycle. If you have \$480 to spend on your new bicycle, how many days can it take Bicycle City to build the bike?

XVIII. Divisibility and Factoring

Summary of the "divisibility rules" to determine factors of other numbers. Use this chart to help with factoring number problems on the next page.

Number	Divisibility Rule	Example
Two (2)	A number is divisible by two if it is	642 is divisible by two
	even. Another way to say a word is	because it ends in a
	even is to say it ends in 0, 2, 4, 6 or 8.	two, which makes it an
		even number
Three	A number is divisible by three if the	423 is divisible by three
(3)	sum of the digits adds up to a	because $4 + 2 + 3 = 9$.
	multiple of three.	Since nine is a multiple
		of three (or is divisible
		by three), then 423 is
		divisible by three
Four (4)	A number is divisible by four if it is	128 is divisible by four
	even and can be divided by two twice.	because half of it is 64
		and 64 is still divisible
		by two
Five (5)	A number is divisible by five if it ends	435 is divisible by five
	in a five or a zero.	because it ends in a five
Six (6)	A number is divisible by six if it is	222 is divisible by six
	divisible by both two and three.	because it is even, so it
		is divisible by two and
		its digits add up to six,
		which makes it divisible
		by three
Nine (9)	A number is divisible by nine if the	9243 is divisible by
	sum of the digits adds up to a	nine because the sum of
	multiple of nine. This rule is similar to	the digits adds up to
	the divisibility rule for three.	eighteen, which is a
		multiple of nine
Ten (10)	A number is divisible by ten if it ends	730 is divisible by ten
	in a zero. This rule is similar to the	because it ends in zero
	divisibility rule for five.	

For each given number, determine the numbers by which the original number is divisible. In the first example, the given number is 10, which is divisible by 2, 5 and 10.

Number			Div	isibl	e By:			
Example: 10	2	3	4	5	6	9	10	
15	2	3	4	5	6	9	10	
27	2	3	4	5	6	9	10	
36	2	3	4	5	6	9	10	
16	2	3	4	5	6	9	10	
28	2	3	4	5	6	9	10	
57	2	3	4	5	6	9	10	
102	2	3	4	5	6	9	10	
268	2	3	4	5	6	9	10	
4518	2	3	4	5	6	9	10	
93	2	3	4	5	6	9	10	
144	2	3	4	5	6	9	10	
256	2	3	4	5	6	9	10	
75	2	3	4	5	6	9	10	
450	2	3	4	5	6	9	10	
70	2	3	4	5	6	9	10	

Find the GCF and the LCM using prime factorization

1. 45, 75	2 . 30, 100	3 . 150, 225

. 36, 54

. 30, 36, 75

. 50, 56, 105