



Preparing for AP Calculus AB - Summer Review Packet – 2022

DUE AUGUST 31, 2022 in the appropriate Google Classroom

Expectations of the Summer Packet:

The problems in this packet are designed to help you review topics that are important to your success in AP Calculus. Remember, the College Board will NOT give you any formulas for the AP Calculus Exam.

All work must be neatly shown for each problem. The problems should be done correctly, not just attempted.

There may be a QUIZ on these concepts at the beginning of school.

All work should be completed and ready to turn in on time.

Some helpful resources:

www.khanacademy.com

www.profrobbob.com (Tarrou's Chalk Talk)

Enjoy your summer!

WELCOME TO AP Calculus AB

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Section 1. Simplify the Rational Expression

Problem	Answer
1. $\frac{1}{x} + \frac{1}{y}$	
2. $\frac{1}{x} + \frac{1}{x^2}$	
3. $\frac{\frac{1}{x}+1}{\frac{1}{x}}$	
4. $\frac{\frac{x}{x+y}}{x}$	
5. $\frac{\frac{1}{x+h} + \frac{1}{x}}{x}$	

Section 2. Factoring

Factor and simplify the expression completely.

If it cannot be factored, write “prime”

Problem	Answer
6. $x^2 - 16$	
7. $x^2 - x - 6$	
8. $6x^2 - x - 2$	
9. $4x^3 - 19x^2 - 5x$	
10. $x^2 + 9$	
11. $x^4 - 13x^2 - 30$	
12. $x^3 + 27$	
13. $x^3 - 8$	
14. $(2x - 3)^3(x + 1) + (x - 3)(2x - 3)^2$	
15. $(3x - 2)^{-4}(x + 3) + (x + 3)^2(3x - 2)^{-3}$	

Section 3. Solving Equations

Solve each equation to find the value of x

Problem	Answer
16. $x^2 + 5x - 24 = 0$	
17. $x^2 - 9 = 5$	
18. $3x^2 - 5x - 2 = 0$	
19. $x^2 - 4x = 0$	
20. $(x - 1)(x^2 - 11x + 30) = 0$	
21. $\sqrt{x} + 1 = 41$	
22. Solve for x $\frac{y}{x+1} = \frac{z}{x}$	
23. $\sqrt[3]{x+1} - 4 = -1$	
24. $x^{-2} = \frac{1}{9}$	
25. $2\sqrt{x} = x - 3$	

Problem	Answer
26. $\frac{8+x}{x} - 5 = 0$	
27. $x^{-1} = -3$	
28. $x^{\frac{4}{3}} = 81$	
29. Solve the inequality $3x^2 - 6x - 24 \leq 0$	
30. $\frac{2x-1}{(x+2)(x^2+3)} = 0$	
31. $x^3 - 2x^2 - 5x + 10 = 0$	

Section 4. Logarithmic Functions

Which of the following are true? Check the box for True or False.

Problem	TRUE	FALSE
32. $2 \ln 2 = \ln 4$		
33. $\frac{\ln 8}{\ln 2} = \ln 4$		
34. $\ln 8 - \ln 2 = \ln 4$		
35. $\ln 4 + \ln 1 = \ln 4$		
36. $\ln 4 \cdot \ln 1 = \ln 4$		
37. $(\ln 2)^2 = \ln 4$		

Section 5. Equations of Lines

Slope-intercept form $y = mx + b$

Point-slope form $y - y_1 = m(x - x_1)$

Problem	Answer
38. Find the equation of the line that has a slope of 5 and passes through the point $(3, -4)$	
39. Find the equation of the line that passes through the points $(4, 1)$ and $(3, -2)$	
40. Find the equation of the line that passes through the points $(-2, 1)$ and is parallel to the line $4x + 2y = -1$	
41. Find the equation of the line that has a slope of 0 and passes through the point $(-5, 1)$	
42. Find the equation of the line that passes through the origin and is perpendicular to the line $3x + 4y = -7$	
43. Find the equation of the line that has an undefined slope and passes through the point $(4, -5)$	

Section 6. Intercepts

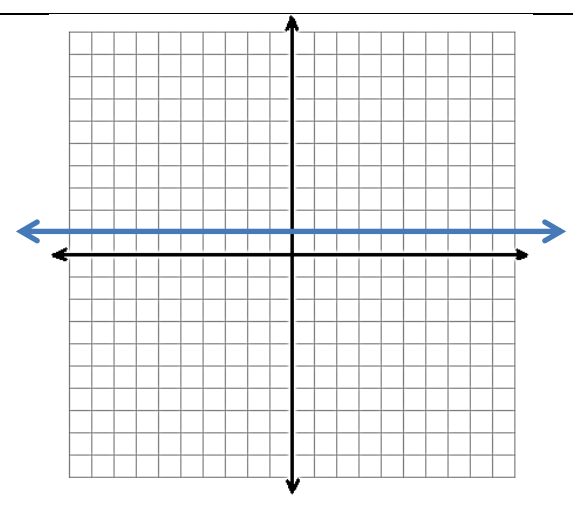
Problem	Answer
43. Find the x and y intercepts of $x^2 + y^2 = 9$	x-intercepts: y-intercepts:
45. Find the equation of the line that has an x-intercept of 5 and a y-intercept of 3.	

Section 7. Graphing equations and functions

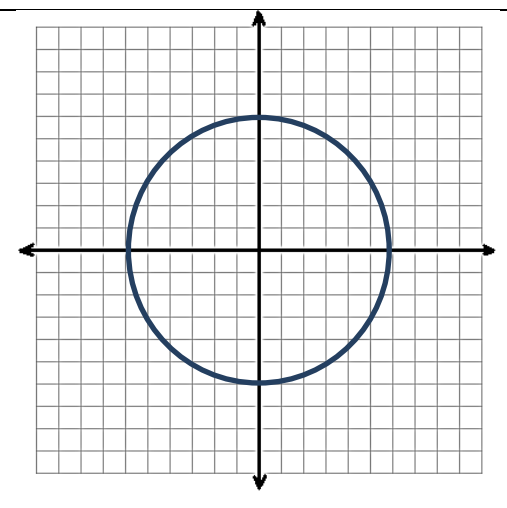
Write the equation of the given graph.

	Graph	Equation
46.		
47.		
48.		

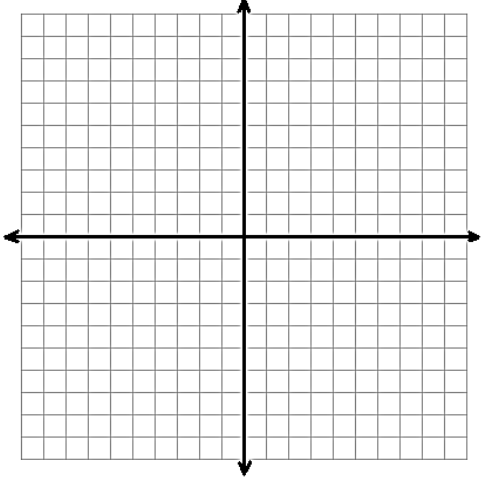
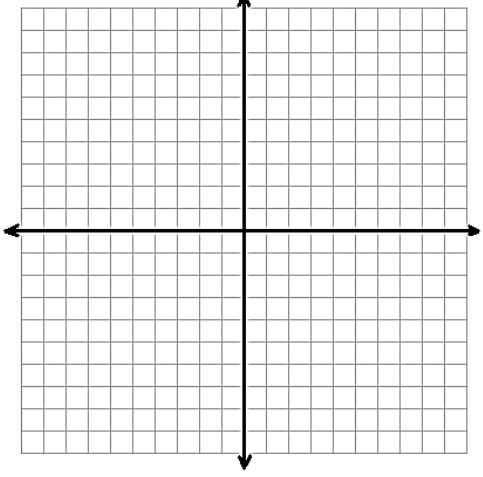
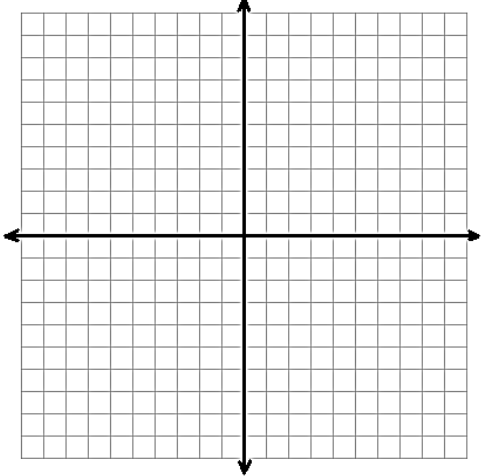
49.



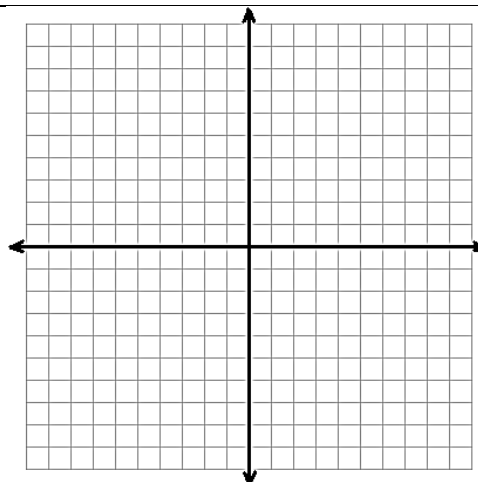
50.



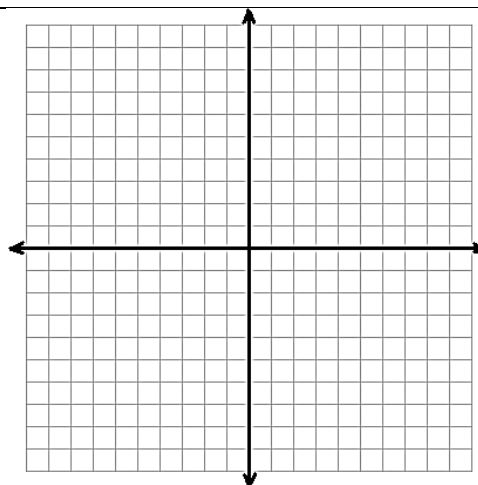
Sketch the graph of the following equations and functions WITHOUT using a graphing calculator. For example, make a table of values.

51. $f(x) = 3x + 1$	
52. $f(x) = x^2$	
53. $f(x) = x $	

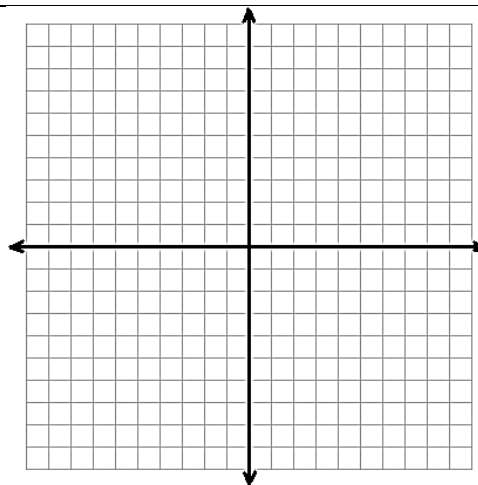
54. $f(x) = x^3$



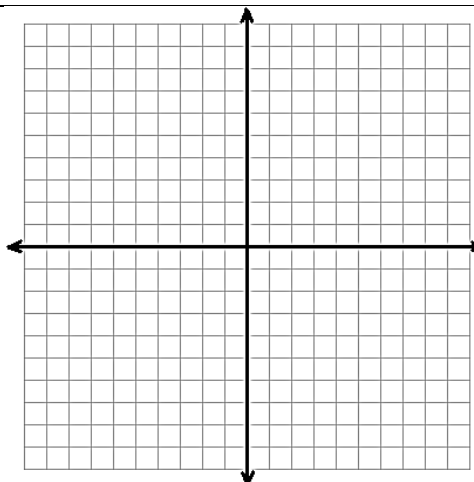
55. $x = 3$



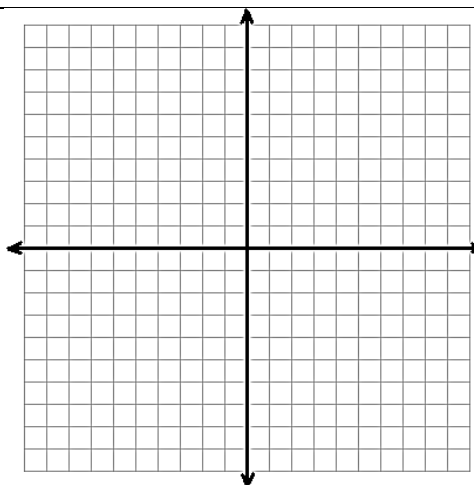
56. $y = -4$



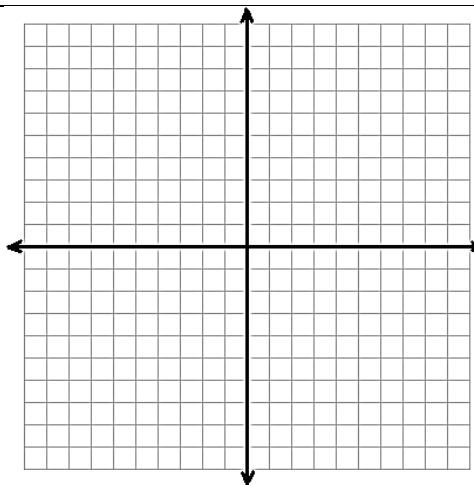
57. $f(x) = \ln x$



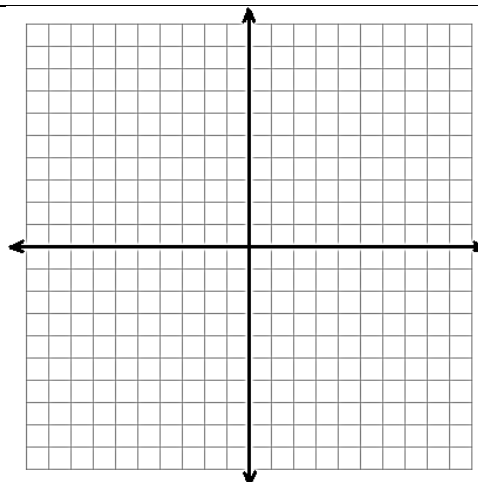
58. $f(x) = \sqrt{x}$



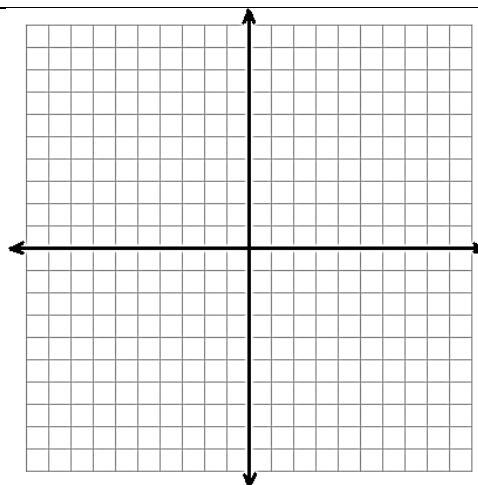
59. $f(x) = \frac{1}{x}$



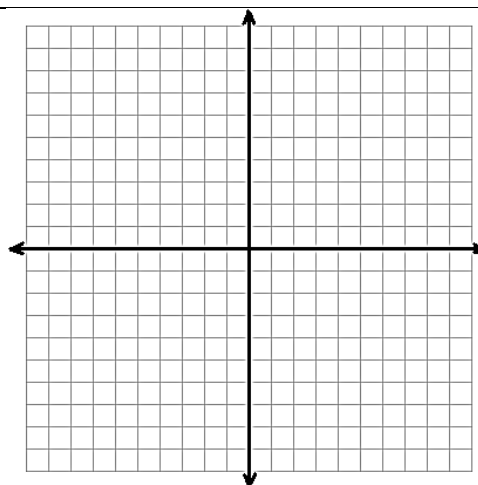
60. $f(x) = \frac{1}{x^2}$



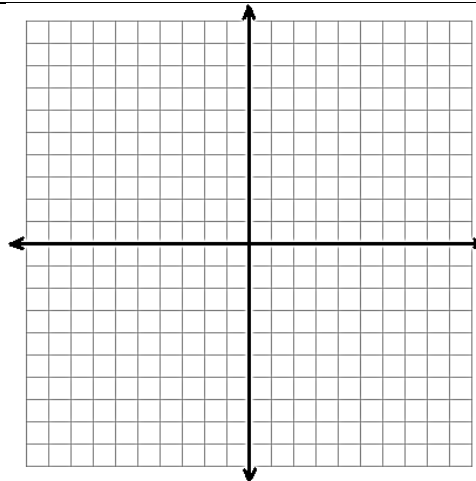
61. $f(x) = |x + 1|$



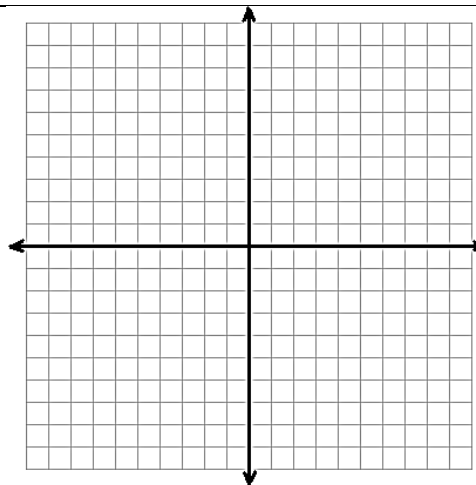
62. $f(x) = x^2 + 2x - 3$



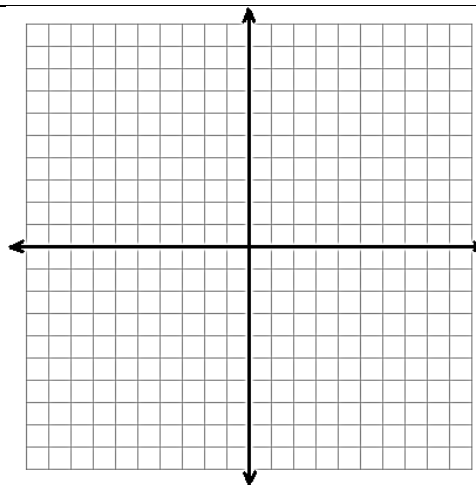
63. $f(x) = x^3 + 1$



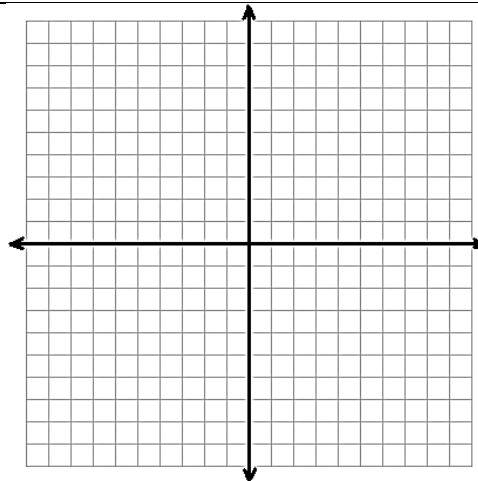
64. $f(x) = (x + 1)^2$



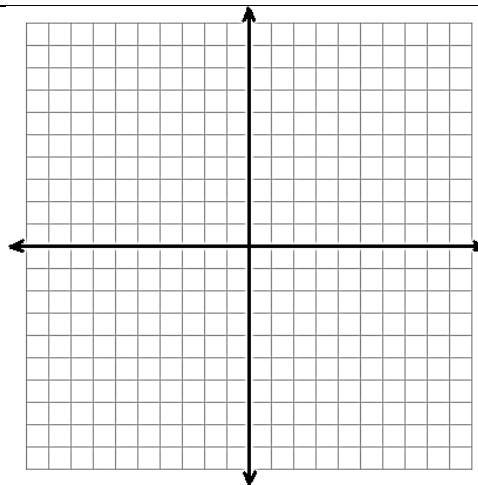
65. $f(x) = -x^2 + 1$



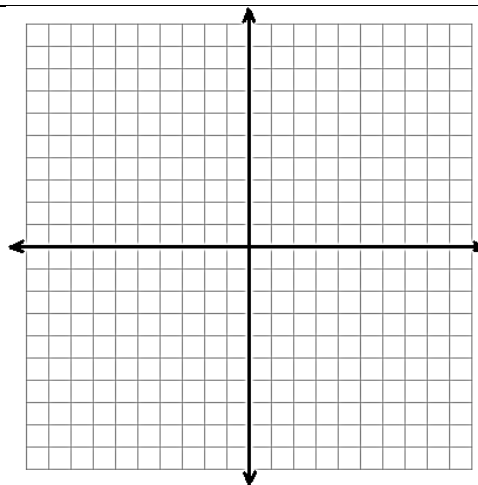
66. $f(x) = (x + 1)^{\frac{1}{3}}$



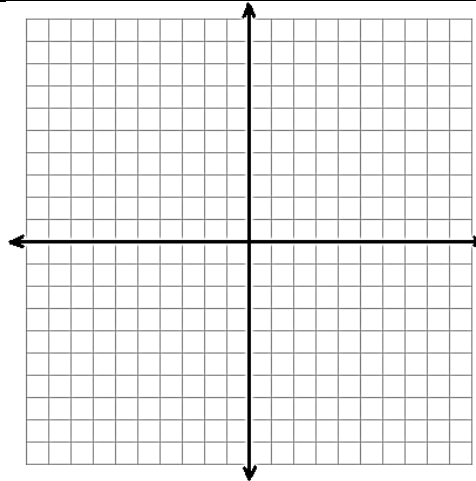
67. $f(x) = x^{\frac{2}{3}}$



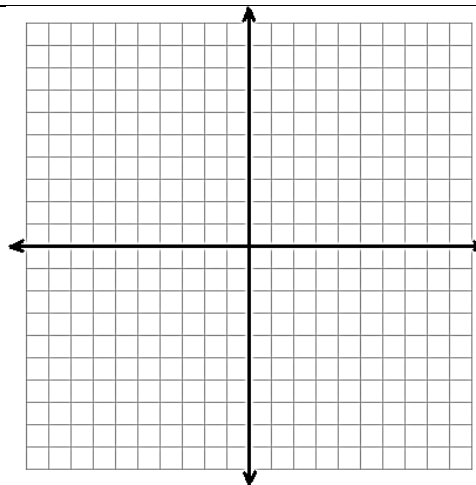
68. $f(x) = e^x$



69. $f(x) = -\frac{1}{x}$



70. $x^2 + y^2 = 25$



Section 8. Domain and Range

Find the Domain and Range of each function

Problem	Answer
71. $f(x) = x - 1$	Domain: Range:
72. $f(x) = \frac{1}{x}$	Domain: Range:
73. $f(x) = \frac{1}{x^2+1}$	Domain: Range:
74. $f(x) = e^x$	Domain: Range:
75. $f(x) = \sqrt{x - 4}$	Domain: Range:
76. $f(x) = x - 1 + 2$	Domain: Range:
77. $f(x) = \ln x$	Domain: Range:
78. $f(x) = \sqrt{x^2 - 3x - 4}$	Domain: Range:
79. $f(x) = \frac{1}{x+6} - 10$	Domain: Range:

Section 9. Inverses

Find the inverse of each function

Problem	Answer
80. $f(x) = x + 3$	
81. $f(x) = \sqrt{x}$	
82. $f(x) = \frac{x}{x+2}$	
83. $f(x) = \ln x$	

Section 10. Compositions of Functions

Use the following functions to evaluate the compositions.

$$f(x) = x^3 + 1 \quad g(x) = x^2 - 2 \quad \text{and} \quad j(x) = x + 3$$

Problem	Answer
84. $f(2)$	
85. $f(j(x))$	
86. $f(j(2))$	
87. $g(g(x))$	
88. $f(x + h)$	
89. $\frac{f(x+h)-f(x)}{h}$	

Section 11. Systems of Equations

Solve the system of equations using any method (substitution, elimination, Cramer's rule, etc.)

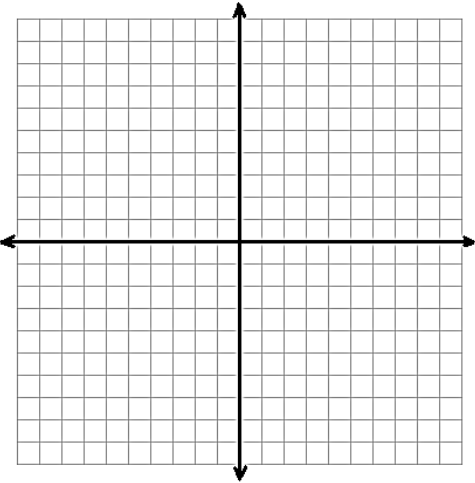
Problem	Answer
90. $2x + 3y = 8$ $x + 2y = 5$	
91. $y = x^2 + 2x + 9$ $7x + y = 19$	

92. The length, l , of a certain rectangle is twice the width, w .

a) Write an equation for the perimeter of the rectangle as a function of its width.

b) If the area of the rectangle is 50 square feet, find the length and width of the rectangle.

Section 12. Intersection of Curves

Problem	Answer
93. Find the point of intersection between the lines $y = x + 1$ and $3y - x = 5$	
94. Find the point of intersection between the line $y = x + 7$ and the curve $y = x^2 + 2x + 5$. Also sketch the area between the graphs.	 A blank coordinate grid for sketching the area between the line $y = x + 7$ and the curve $y = x^2 + 2x + 5$. The grid is 20 units wide and 15 units high, with a horizontal x-axis and a vertical y-axis intersecting at the center. Both axes have arrows at their ends.

Section 13. Formulas

Write the specified formula (without looking it up). You will need to know these because the College Board provides NO formulas.

Problem	Formula
95. Quadratic formula	
96. Pythagorean Theorem	
97. the hypotenuse of a 45-45-90 right triangle with a leg length of x	
98. the hypotenuse of a 30-60-90 right triangle with the shortest leg length of x	
99. Volume of a Sphere	
100. Volume of a Cylinder	
101. Volume of a Cone	
102. Volume of a box with a square base x and height h	
103. Surface Area of a Sphere	
104. Surface Area of a Cylinder with NO top.	

Problem	Formula
105. Area of a Triangle	
106. Area of a Trapezoid	
107. Cross Section through the center of a Sphere	
108. Area of an equilateral triangle in terms of the length of a side s	

Section 14. Similar Triangles

Problem	Answer
109. A six-foot tall man is standing 10 feet away from a 20 foot lamppost. What is the length of his shadow? Draw a picture.	
110. Water is dripping out of a conical tank that has a diameter of 8 inches and a height of 12 inches. When the depth of the water is 8 inches, what is the radius of the conical tank? Draw a picture.	

Section 15. Horizontal and Vertical Asymptotes

Asymptotes are lines that a graph approaches.

For $f(x) = \frac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ have no common factors other than one, and n is the degree of $a(x)$ and m is the degree of $b(x)$:

- **Vertical asymptotes:** May occur at the real zeros of $b(x)$. (Set the denominator = 0)
- **Horizontal asymptotes:**
 - If $n < m$, the asymptote is $y = 0$.
 - If $n = m$, the asymptote is $y = c$, where c is the ratio of the leading coefficients of the numerator and denominator.
 - If $n > m$, there is no horizontal asymptote.
- **Oblique asymptotes:** If $n = m + 1$, the equation is the quotient polynomial $q(x)$ of $f(x)$, or
$$\frac{a(x)}{b(x)} = q(x) + \frac{r(x)}{b(x)}.$$
- x -intercepts, if any, occur at the real zeros of $a(x)$. The y -intercept, if it exists, is the value of f when $x = 0$.

Find the equations of the horizontal and vertical asymptotes of each function.

Problem	Answer
111. $y = \frac{1}{x-1}$	Horizontal Asymptote: Vertical Asymptote:
112. $y = \frac{x^3}{x^3-1}$	Horizontal Asymptote: Vertical Asymptote:

Section 16. Exponents and Logarithms

Which of the following are true? Check the box for True or False.

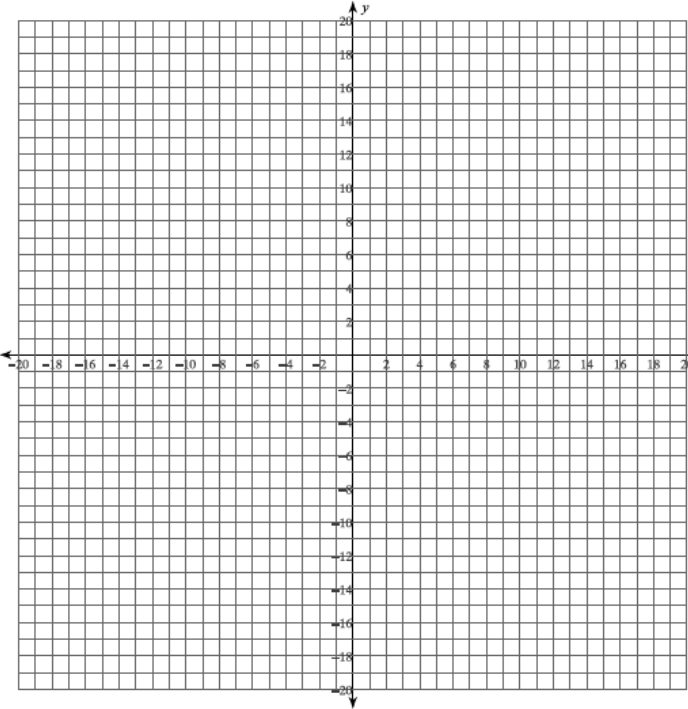
Problem	TRUE	FALSE
113. $x^0 = 1$		
114. $x^{-2} = \frac{1}{x^2}$		
115. $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$		
116. $x^5 \cdot x^3 = x^{15}$		
117. $x^5 \cdot y^5 = (xy)^5$		
118. $(x^3)^5 = x^8$		
119. $x^{5-w} = \frac{x^5}{x^w}$		
120. $x^{t+5} = (x^t)^5$		
121. $\sqrt{\frac{9}{4}} = \frac{3}{2}$		
122. $(4x)^{\frac{1}{2}} = 2x$		

Problem		
123a. $\sqrt{\frac{1}{x}} = x^{-\frac{1}{2}}$		
123b. $\sqrt{x^2} = x$		
124. $\sqrt{x^2 - 25} = x - 5$		
125. $x^{\frac{4}{3}} = \sqrt[4]{x^3}$		
126. $(x^{\frac{1}{2}} + y^{\frac{1}{2}})^2 = x + y$		
127. $x^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{x^2}}$		
128. $e^{\ln x^2} = x^2$		
129. $\ln e^3 = 3$		
130. $e^{2 \ln 2 - \ln 5} = \frac{4}{5}$		
131. $\ln x^2 = (\ln x)^2$		

Section 17. Using the Graphing Calculator

Make sure you know how to use your graphing calculator. Use the functions in the graphing calculator (“calc” feature over the “trace” button at the top)

Write your answers to 3 decimal places.

Problem	Answer
<p>132. Graph the function on the xy-plane</p> $y = 0.1x^3 + 2x^2 - x - 3$	
<p>133. Find the roots of the equation above.</p>	
<p>134. Find the point of intersection for the graphs</p> $y = x^3 + x - 3$ <p style="text-align: center;">and</p> $y = 2x + 4$	

135. Find the maximum value for the graph $f(x) = -x^4 + x - 4$	
136. For the function in #135, find the interval(s) on which $f(x)$ is increasing.	

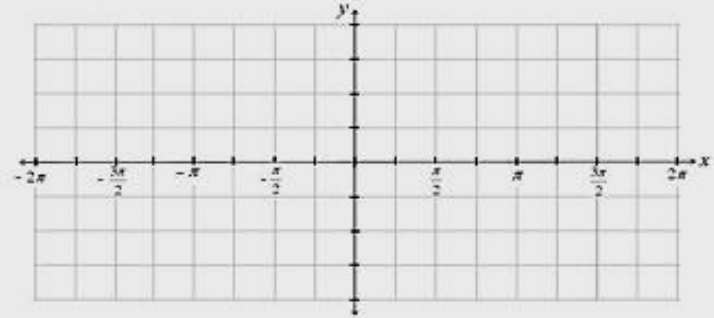
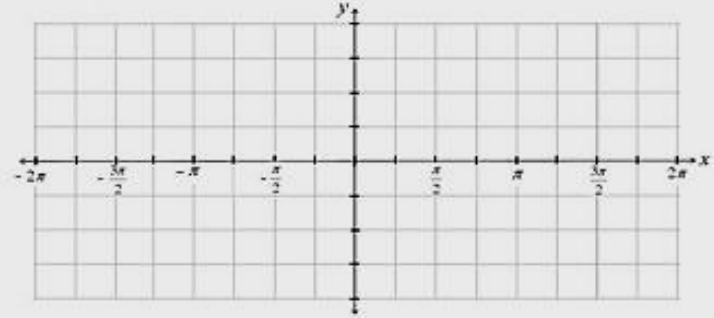
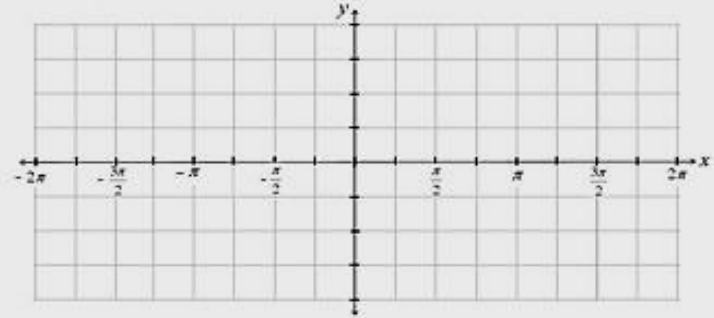
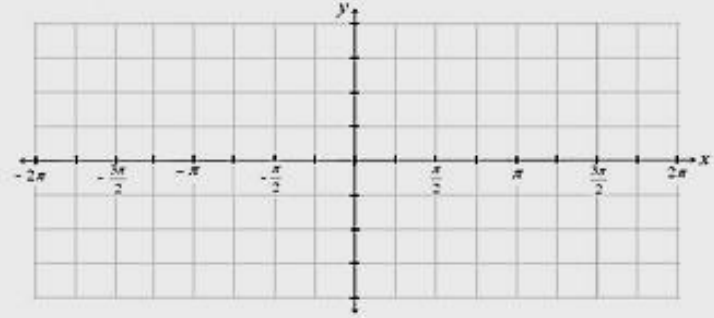
Section 18. Trigonometric Functions and Identities

Evaluate the following expressions WITHOUT using the Unit Circle, any tables, or your calculator. Spend some time memorizing the Unit Circle first, then try the problems. If the function is undefined, write “undefined”.

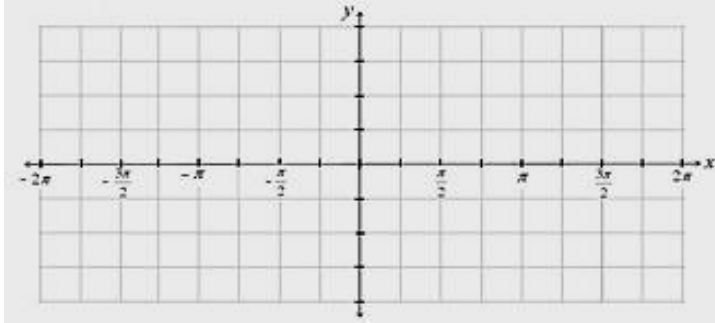
Problem	Answer
137. $\sin\left(\frac{\pi}{6}\right)$	
138. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$	
139. $\tan\left(\frac{7\pi}{6}\right)$	
140. $\cos(0)$	
141. $\cos\left(\frac{\pi}{4}\right)$	
142. $\csc\left(\frac{-5\pi}{6}\right)$	
143. $\sec(\pi)$	
144. $\cot\left(\frac{-\pi}{2}\right)$	

145. $\sin^{-1}\left(\frac{1}{2}\right)$	
146. $\tan\left(\frac{\pi}{2}\right)$	
147. $\sin^2\left(\frac{5\pi}{6}\right)$	
148. $\cot\left(\frac{2\pi}{3}\right)$	
149. $\sin\left(\frac{\pi}{2}\right)$	
150. $\cot^{-1}(-1)$	
151. $\sec\left(\frac{3\pi}{4}\right)$	
152. $\tan^{-1}(-1)$	
153. $\csc(\pi)$	
154. $\sec^2\left(\frac{\pi}{4}\right)$	

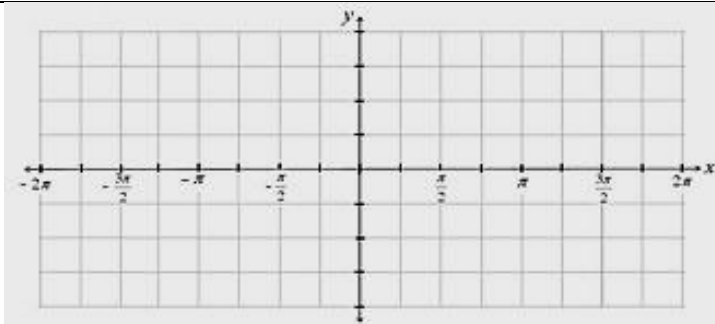
Sketch only **one period** of the following trigonometric graphs.

Problem	Answer
155. $y = \sin x$	
156. $y = \cos x$	
157. $y = \tan x$	
158. $y = \sec x$	

159. $y = \csc x$



160. $y = \cot x$



Solve the following trigonometric equation on $[0, 2\pi]$

Problem	Answer
161. $\sin x = \cos x$	